CHURCHLANDS
SENIOR HIGH SCHOOL

MATHEMATICS SPECIALIST 3,4
TEST 2 SECTION ONE 2016
NON Calculator Section
Chapters 3 and 4

Name______________________________

Time: 35 minutes
Total: 35 marks

Question 1

(7 marks)

Two functions are defined as $f(x)=\sqrt{x-1}$ and $g(x)=\frac{1}{x-1}$

(a) Evaluate $gf\left(\frac{13}{9}\right)$ (2 marks)

$$= g\left(\sqrt{\tfrac{13}{9}-1}\right)$$
$$= g\left(\sqrt{\tfrac{4}{9}}\right)$$
$$= g\left(\tfrac{2}{3}\right) \quad ✓$$
$$= \frac{1}{\frac{2}{3}-1} = -3 \quad ✓$$

(b) Find in simplified form $gg(x)$. (2 marks)

$$g\left(\frac{1}{x-1}\right) = g\left(\frac{1}{\frac{1}{x-1}-1}\right) \quad ✓$$
$$= \frac{1}{\frac{1-(x-1)}{x-1}} = \frac{x-1}{2-x} \quad \left\{\frac{x}{1-x}\right\}$$

(c) Determine the domain of $f(g(x))$ (3 marks)

$$= \sqrt{\frac{1}{x-1}-1} \quad \text{need} \quad \frac{1}{x-1}-1 \geq 0$$
$$\frac{1-(x-1)}{x-1} \geq 0$$
$$\frac{2-x}{x-1} \geq 0 \quad ✓$$

$\therefore$ Domain $1 < x \leq 2$

✓ ✓

Question 2 **(6 marks)**

(a) Determine the domain and range of $f(g(x))$ given that $f(x) = \dfrac{12}{x+1}$ and $g(x) = \sqrt{x+1}$ (3)

$$f\,g(x) = \frac{12}{\sqrt{x+1}\ +1}$$ ✓

$D : \{x : x \in R \,.\, x \geq -1\}$ ✓

$R : \{y : y \in R\ 12 \geq y \geq 0\}$ ✓

(b) Given that $f(x) = 2x+3$ and $g(f(x)) = 4x^2+12x+11$, find $g(x)$. (3)

Let $k = 2x+3 \Rightarrow x = \dfrac{k-3}{2}$

$$g(k) = 4\left(\frac{k-3}{2}\right)^2 + 12\left(\frac{k-3}{2}\right) + 11$$ ✓

$$= 4\left(\frac{k^2-6k+9}{4}\right) + 6(k-3) + 11$$ ✓

$$= k^2 - 6k + 9 + 6k - 18 + 11$$

$$= k^2 + 2$$

$$\therefore g(k) = \underline{k^2 + 2}$$ ✓

$$\begin{array}{r} 2x+3 \\ 2x+3\overline{\smash{)}\,4x^2+12x+11} \end{array}$$

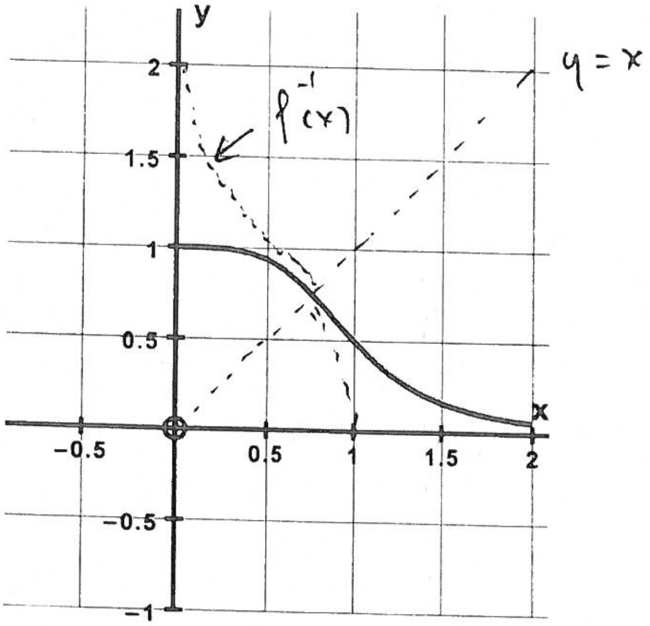
$\underline{\underline{r = 2}}$

ie square it then add $\underline{2}$

Question 3

(6 marks)

The graph of function $f(x) = \frac{1}{x^4+1}$ for the domain $0 < x < 2$ is shown below.



(a) Determine the exact value for $\lim_{x \to 2^+} f(x)$ $= \frac{1}{2^4+1}$ ✓ $= \frac{1}{17}$ ✓ (2)

(b) On the axes given above, sketch the graph of the inverse function, $y = f^{-1}(x)$ (2)

✓1 Reflect in $y = x$

✓1 pt of intersect at $\underline{\underline{x = y}}$

(c) Obtain the rule for $f^{-1}(x)$. (2)

$f^{-1}(x) = \sqrt[4]{\frac{1}{x} - 1}$ or

$$y = \frac{1}{x^4+1}$$

$$x = \frac{1}{y^4+1}$$

$$y^4 + 1 = \frac{1}{x}$$

$$y^4 = \frac{1}{x} - 1$$

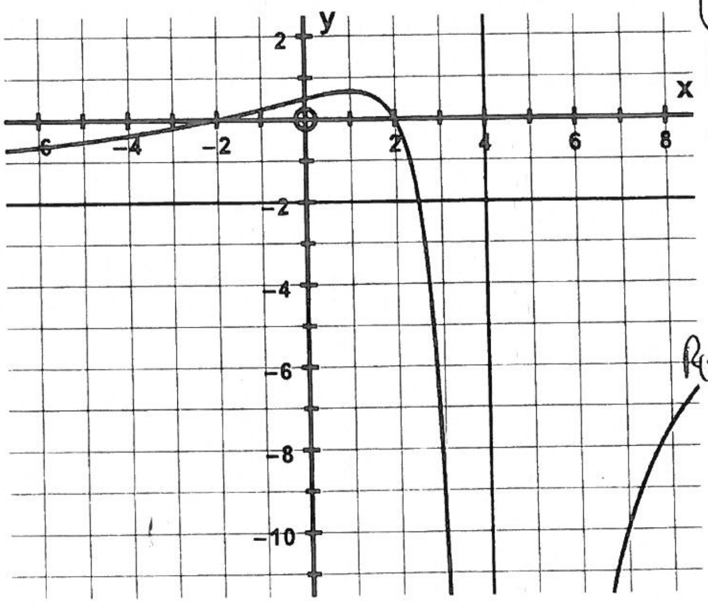
$$y = \sqrt[4]{\frac{1}{x} - 1}$$

$$\left\{ y = \sqrt[4]{\frac{1-x}{x}} \right\}$$

Question 4 **(5 marks)**

A rational function $R(x)$ is sketched below. Function $R(x)$ has the following properties:

- Only one pole or a discontinuity at $x = 4$
- Two horizontal intercepts at $x = 2$ *and* $x = -2$.
- A horizontal asymptote at $y = -2$



For a

$k(x^2-a)=0$ for $R(x)=0$

$\Rightarrow x^2-a=0$

this occurs when

$x=+2$ or -2 intercepts

$2^2-a=0 \Rightarrow a=\underline{4}$ ✓

$\downarrow$

$R(x)=\dfrac{k(x^2-4)}{(x-b)(x-c)}=\dfrac{k(x-2)(x+2)}{(x-b)(x-c)}$

Discont at $x=4$ only ✓

$\Rightarrow$ Denominator

$(x-b)(x-c)$ $\underline{b=c=4}$

(a) If $R(x)=\dfrac{k(x^2-a)}{(x-b)(x-c)}$ explain why $k=-2,\ a=4, b=4$ *and* $c=4$

$\lim\limits_{x\to\infty} R(x) = \underline{\underline{-2}}$ ✓ considering dominant terms

$\therefore \dfrac{kx^2-\cancel{ka}}{x^2-\cancel{xb}-\cancel{xc}+\cancel{bc}} \approx \dfrac{k\cancel{x^2}}{\cancel{x^2}}$

$\approx \underline{k=-2}$

$x=\pm 2$ → intercepts,

so $x^2-a=$ ~~$(x+a)(x-a)$~~ 0

diff of squares

so $\cancel{a}^2=\cancel{4} \Rightarrow a \pm 4$

(4)

OR. when $x=3$ $y=-10$

$\therefore \dfrac{5k}{-1}=-10 \Rightarrow k=-2$

(b) Determine $\lim\limits_{x\to 4} R(x)$. Does not exist.

(1)

Question 5

(7 marks)

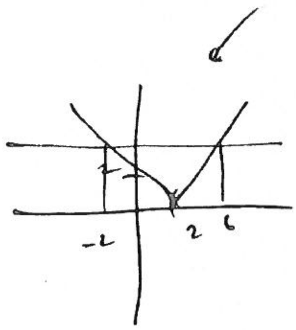
Solve the following.

(a) $|x-2|>4$

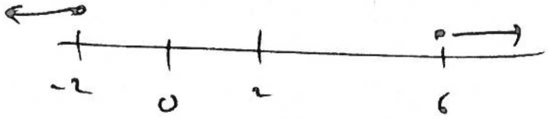
(1)

$x - 2 = 4 \Rightarrow \underline{x = 6}$

$-x + 2 = 4 \Rightarrow \underline{x = -2}$ ✓ ∴



or dist fr. 2 to any pt is greater than 4



or

$(x-2)^2 = 4^2$

$x^2-4x+4=16$

$x^2-4x-12=0$

(x [illegible]6)(x+2) = 0

$\therefore x = +6$

$\underline{x = -2}$ ✓

$\therefore \underline{x>6, \; x<-2}$

(b) $|x-7| \le |x-11|$

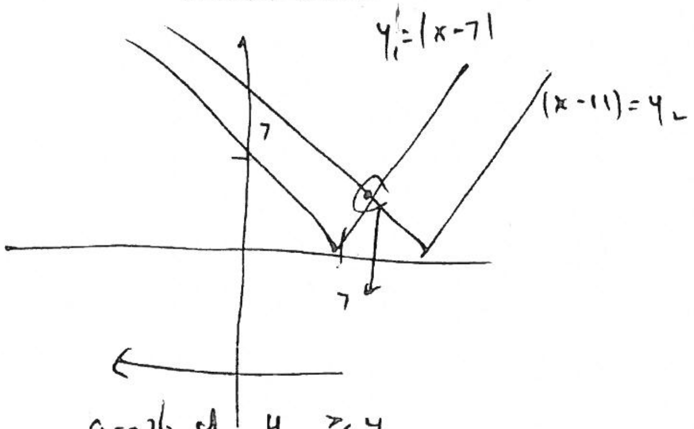
(2)

$(x-7)^2 \le (x-11)^2$

$x^2 - 14x + 49 \le x^2 - 22x + 121$

$8x \le 72$

$\underline{x \le 9}$



graph of $y_2 \geq y_1$

solve $-(x-11) \geq x-7$

$-x+11 \geq x-7$

$18 \geq 2x$

$\underline{9 \geq x}$

(c) $|3x+4| \geq |5x+2|$ (2)

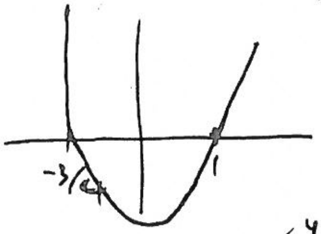
$(3x+4)^2 \geqslant (5x+2)^2$

$9x^2 + 24x + 16 \geqslant 25x^2 + 20x + 4$

$0 \geqslant 16x^2 - 4x - 12$

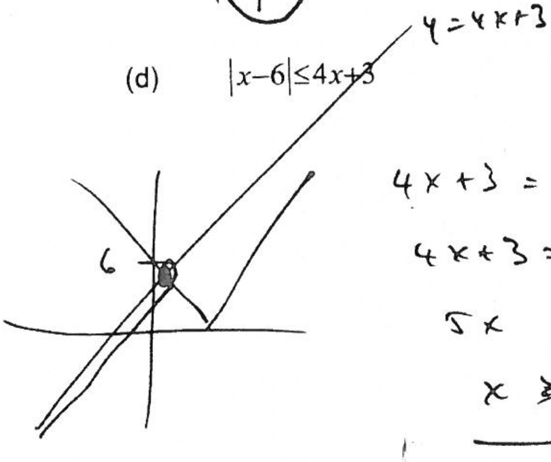
$0 \geqslant 4x^2 - x - 3$

$0 \geqslant (4x+3)(x-1)$



$-\frac{3}{4} \leq x \leq 1$

(d) $|x-6| \leq 4x+3$ (2)



$4x+3 = -(x-6)$

$4x+3 = -x+6$

$5x = 3$

$x \geq 3/5$ /

Issues!!

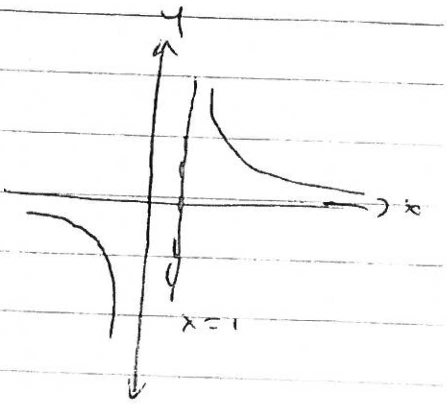
$x^2 - 12x + 36 \leq 16x^2 + 24x + 9$

$0 \leq 15x^2 + 36x - 27$

$0 \leq 5x^2 + 12x - 9$

$(5x-3)(x+3)$

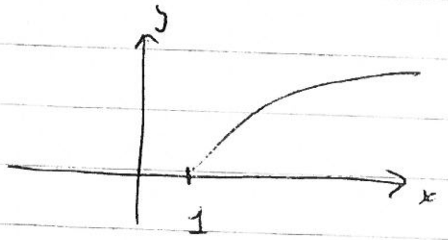
$fg(x)$



now $g(x) = \frac{1}{x-1}$

$D_g : \{x \in R : x \neq 1\}$

$R_g : \{y \in R : y \neq 0\}$



$f(x) = \sqrt{x-1}$

$D_g : \{x \in R : x \geq 1\}$

now the range of $g(x)$ will be the domain of

$\underline{f \text{ in } fg(x)}$

so f cannot ~~has~~ has a natural Domain $x \geq 1$

But $g(x)$ cannot output 1 as $y \neq 1$

so x must be either >1 or <1

Now $fg(x) = \sqrt{\frac{1}{x-1} - 1}$

Note $\frac{1}{x-1} - 1 \geq 0$

$\frac{1}{x-1} \geq 1$

~~x-1~~ $1 \geq x-1 \Rightarrow \underline{\underline{2 \geq x}}$

CHURCHLANDS
SENIOR HIGH SCHOOL

MATHEMATICS SPECIALIST 3,4
TEST 2 SECTION TWO 2016
Calculator Section
Chapters 3 and 4

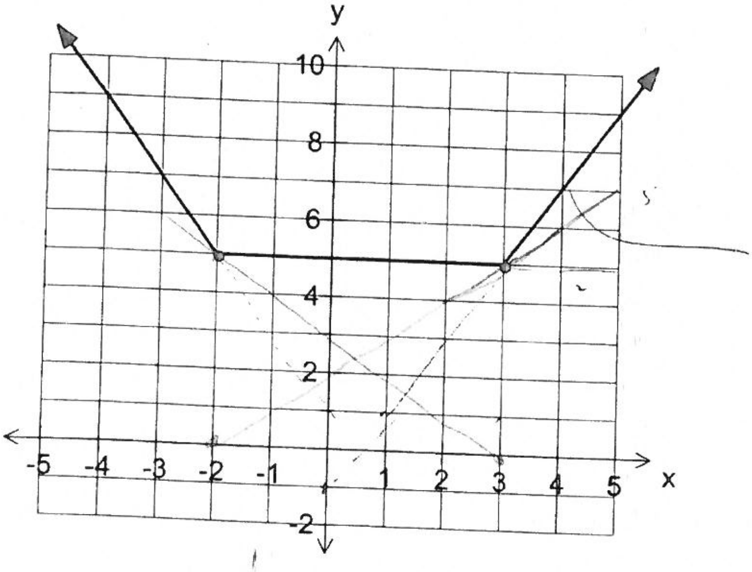
Name

Time: 20 minutes
Total: 20 marks

Question 1

(5 marks)

The function *f*, defined for all real *x* by $f(x) = |x-a| + |x+b|$, where *a* and *b* are positive integers, has the following graph.



$a = 3$

$b = 2$

$2x + c = y$

$6 + c = 5$

$c = -1$

(a) Find the values of *a* and *b*.

$a = 3 \quad b = 2$ ✓✓

$f(x) = |x - 3| + |x + 2|$

(b) Express $f(x)$ as a piecewise function.

$$f(x) = \begin{cases} -2x + 1, & x < -2 \checkmark \\ 5 & -2 \leq x \leq 3 \checkmark \\ 2x - 1 & x > 3 \checkmark \end{cases}$$

Question 2 **(5 marks)**

At 10.00am, two bumper cars at the royal show, G and T, have position vectors, $\underline{r}$ m, and velocity vectors, $\underline{v}$ m/s, as shown below:

$\mathbf{r}_G = 3\mathbf{i} + 9\mathbf{j}$ $\mathbf{v}_G = -\mathbf{i} - \mathbf{j}$

$\mathbf{r}_T = 9\mathbf{i}$ $\mathbf{v}_T = -5\mathbf{i} + 5\mathbf{j}$

Prove that the bumper cars will collide if they continue with these velocities and find the time and location of the collision.

$\underset{\sim}{r}_G = 3\underset{\sim}{i} + 9\underset{\sim}{j} + t(-\underline{i} - \underline{j})$ $r_T = (9 - 5t)\underline{i} + (5t)\underline{j}$

$= (3 - t)\underline{i} + (9 - t)\underline{j}$

For coll

$\therefore$ $3 - t = 9 - 5t$ $9 - t = 5t$

$4t = 6$ $9 = 6t$

$\underline{t = 1.5}$ ✓ $\underline{1.5 = t}$ ✓

$\therefore$ collision occurs at $\underline{1.5 \text{ seconds}}$ ✓

Time $\underline{10.00 + 1.5 \text{ seconds}}$ ✓

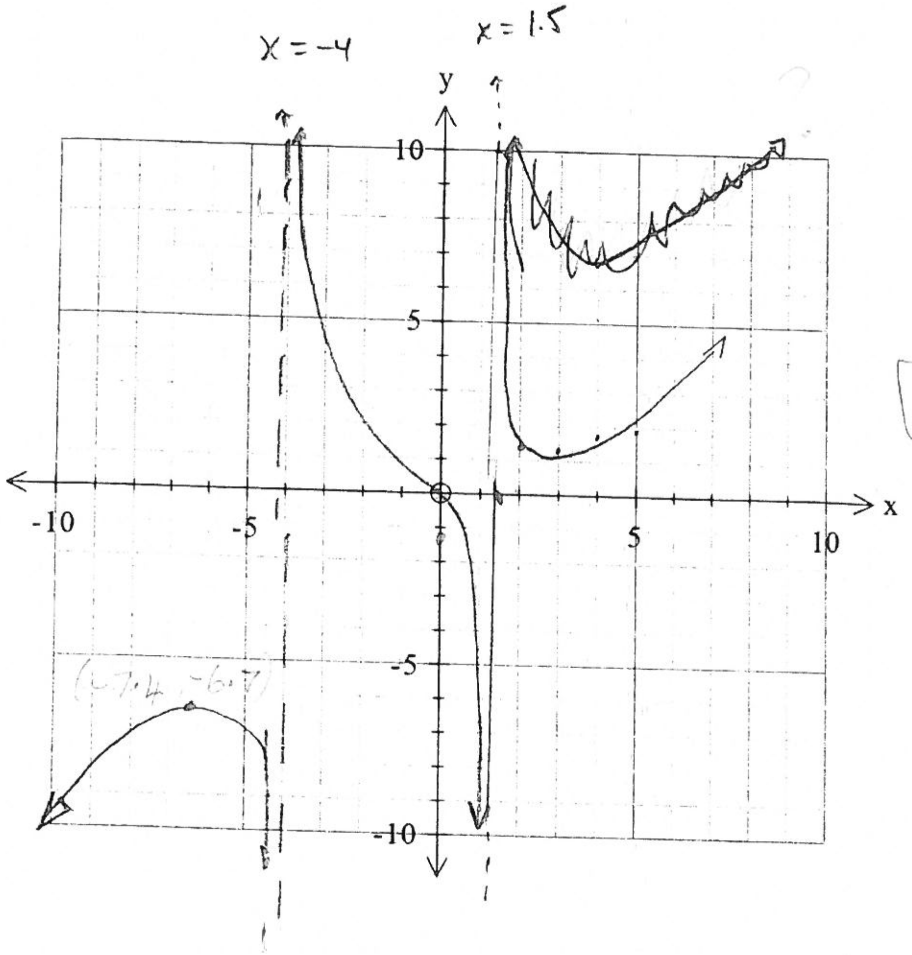
location: $= 9 - 5(1.5)\underline{i} + 5 \times 1.5\underline{j}$

$= 1.5\underset{\sim}{i} + 7.5\underline{j}$ ✓

Question 3

(5 marks)

Sketch the graph $y = \dfrac{x^3}{(x+4)(2x-3)}$., the asymptotes and describe the behaviour of the graph as $x \to \pm\infty$. Give the equations for the vertical and other asymptotes.



asympt ✓

correct shape ✓

correct TPs ✓

$x = -4,\ x = 1.5$ ✓

$\boxed{y = \frac{1}{2}x}$ ✓

(2.4, 1.2)

$$y = \frac{x^3}{2x^2+5x-12}$$

$$2x^2+5x-12 \overline{\smash{)}\ x^3} \quad \frac{1}{2}x - \frac{5}{4}$$

$$x^3 + 2.5x^2 - 6x$$

$$-2.5x^2 + 6x$$

$$-2.5x^2 +$$

as $x \to \pm\infty$ can consider dominant terms.

so $y \simeq \dfrac{\pm x^3}{2x^2}$

$\simeq \pm \dfrac{x}{2}$

$= \pm \dfrac{1}{2}x$

oblique asymptote: $y = \frac{1}{2}x - \frac{5}{4}$

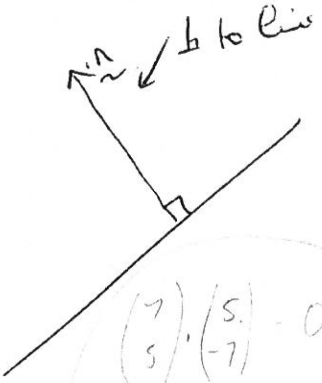
Question 4 **(5 marks)**

Find the Cartesian equation of the line perpendicular to the vector $7\underset{\sim}{i}+5\underset{\sim}{j}$ and passing through the point $(-1,3)$

$\underset{\sim}{r}\cdot\underset{\sim}{n} = \underset{\sim}{a}\cdot\underset{\sim}{n}$ ✓

$\underset{\sim}{r}\cdot(7\underset{\sim}{i}+5\underset{\sim}{j}) = (-1\underset{\sim}{i}+3\underset{\sim}{j})\cdot(7\underset{\sim}{i}+5\underset{\sim}{j})$ ✓

$\underset{\sim}{r}\cdot\begin{pmatrix}7\\5\end{pmatrix} = 8$ ✓



$\begin{pmatrix}7\\5\end{pmatrix}\cdot\begin{pmatrix}5\\-7\end{pmatrix} = 0$

Pt $(-1,3)$
Dir $(5,-7)$

$\underset{\sim}{r} = \begin{pmatrix}-1\\3\end{pmatrix} + \lambda\begin{pmatrix}5\\-7\end{pmatrix}$

now $\underset{\sim}{r} = \begin{pmatrix}x\\y\end{pmatrix}$

$\therefore\ 7x+5y=8$ ✓

$5y=-7x+8$

$y = -\frac{7}{5}x+\frac{8}{5}$ ✓

OR // to line is $-5\underset{\sim}{i}+7\underset{\sim}{j}$

$\therefore\ \underset{\sim}{r} = \langle -1,3\rangle + \lambda\langle -5,7\rangle$

$= \langle -1, -5\lambda\rangle + \langle 3+7\lambda\rangle$

$\therefore\ x = -1-5\lambda \qquad y = 3+7\lambda$

$\lambda = \frac{x+1}{-5} \qquad \lambda = \frac{y-3}{7}$

$\Rightarrow\ 7x+7 = -5y+15$

$\underline{7x+5y=8}$